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WEST BENGAL STATE UNIVERSITY

B.Sc. Honours PART-II Examinations, 2016

MATHEMATICS-HONOURS

PAPER-MTMA-IV

Time Allotted: 4 Hours | Full Marks: 100

The figures in the margin indicate full marks. Candidates should answer in their own words and adhere to the word limit as practicable.

Group-A

Answer any *two* questions from the following: 10×2 = 20

1. (a) Prove that the locus of the pole of normal chord of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is the curve $a^6y^2-b^6x^2=(a^2+b^2)^2x^2y^2$. 5

   (b) Two tangents to the parabola $y^2=4ax$ meet at an angle of 45°. Prove that the locus of the point of intersection is the curve $y^2-4ax=(x+a)^2$. 5

2. (a) The section of a cone whose guiding curve is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ by the plane $x=0$ is a rectangular Hyperbola. Show that the locus of the vertex is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$. 5

   (b) Find the equations to the tangent planes to the conicoid $2x^2-6y^2+3z^2=5$ which pass through the line $x+9y-3z=0=3x-3y+6z-5$. 5

3. (a) Find the equations of the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ through a point of the principal elliptic section. Hence, show that the projection of the generators of a hyperboloid on coordinate planes is tangents to the section of the hyperboloid by that plane. 5

(b) Reduce the equation $6y^2-18yz-6zx+2xy-9x+5y-5z+2=0$ to the canonical form and state the nature of the surface represented by it. 5

Group-B

Answer any *one* question from the following: $10\times1=10$

4. (a) Solve: $\frac{dx}{z}=\frac{dy}{-z}=\frac{dz}{z^2+(x+y)^2}$. 3

(b) Find the complete integral of the equation $p^3+q^3=3pqz$. 3

(c) Apply Charpit's method to find the complete integral of $(p+q)(px+qy)=1$. 4

5. (a) Solve: $z(z-y)dx+z(z+x)dy+x(x+y)dz=0$. 5

(b) Find the eigen values and eigen functions for the differential equation $\frac{d^2y}{dx^2}+\lambda y=0$, which satisfies the boundary conditions $y(0)=0$ and $y(\pi)=0$. 5

Group-C

6. (a) Define a basic feasible solution for a L.P.P: 7

Max $Z=cx$

Subject to $Ax=b$; $x\geq 0$

Prove that if for basic feasible solution X_B of the above L.P.P $z_j-c_j\geq 0$ for every column a_j of A, then X_B is an optimal solution.

(b) Use Charne's M-method to salve the following L.P.P. 6

Maximize $Z=x_1+5x_2$

Subject to $3x_1 + 4x_2 \le 6$

$x_1 + 3x_2 \ge 3$

$x_1, x_2 \ge 0$.

Or

7. (a) Define the dual of the given primal L.P.P. 7

Max $Z = cx$

Subject to $Ax \le b,\ x \ge 0$.

Prove that if the primal problem has an unbounded objective function then the dual has no feasible solution.

(b) Use duality to solve the following L.P.P. 6

Minimize $Z = 4x_1 + 3x_2 + 6x_3$

Subject to $x_1 + x_3 \ge 2$

$x_2 + x_3 \ge 5$

$x_1, x_2, x_3 \ge 0$.

8. (a) Show that the number of basic variables in transportation problem is at most $(m+n-1)$, where m is the number of origins and n is the number of destination. 6

(b) Solve the assignment problem with the following cost matrix 6

	I	II	III	IV	V
A	9	8	7	6	4
B	5	7	5	6	8
C	8	7	6	3	5
D	8	5	4	9	3
E	6	7	6	8	5

Or

9. (a) Prove that if we add a fixed number P to each element of the payoff matrix, then the optimal strategies remains unchanged while the value of the game is increased by P. 6

(b) Use dominance to reduce the payoff matrix and solve the following game problem given by the payoff matrix. 6

B

	–5	3	1	20
A	5	5	4	6
	–4	–2	0	–5

Group-D

Answer any *three* questions from the following: $15\times3 = 45$

10.(a) A particle of unit mass is projected with a velocity u at an inclination α above the horizon in a medium, the resistance of which is k times the velocity. Show that its direction will make an angle $\frac{\alpha}{2}$ with the horizon after a time $\frac{1}{k}\log\left(1+\frac{ku}{g}\tan\frac{\alpha}{2}\right)$. 8

(b) Two perfectly inelastic bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy is 7

$\frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(u_1-u_2)^2$.

11.(a) A particle of mass m is projected vertically under gravity. The resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{v^2}{g}[\lambda-\log(1+\lambda)]$ after a time $\frac{v}{g}\log(1+\lambda)$ where v is the terminal velocity of the particle and λv is its initial velocity. 7

14.(a) An engine is pulling a train and works as a constant power, doing H units of work per second. If M is the mass of the whole train and F, the resistance (supposed constant) then prove that the time of generating velocity V from rest is $\left(\frac{MH}{F^2}\log\frac{H}{H-FV}-\frac{MV}{F}\right)$ seconds. 7

(b) One end of an elastic string, of unstretched length a, is tied to a point on a smooth table and a particle is attached to the other end, and can move freely on the table. If the path be nearly a circle of radius b, then show that its apsidal angle is approximately $\pi\sqrt{\frac{b-a}{4b-3a}}$. 8