WEST BENGAL STATE UNIVERSITY

B.Sc. Honours 1st Semester Examination, 2018

MTMACOR02T-MATHEMATICS (CC2)

ALGEBRA

Time Allotted: 2 Hours | Full Marks: 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

Answer Question No. 1 and any *five* from the rest

1. Answer any *five* questions from the following: $2\times 5 = 10$

(a) Express $z=-1+i\sqrt{3}$ in polar form.

(b) Prove that $2^n > 1+n\sqrt{2^{n-1}}$

(c) Solve $x^7=1$

(d) Find the condition that the roots of the equation $x^3-px^2+qx-r=0$ will be in G.P.

(e) Show that the following equation has at least four imaginary roots.

$$4x^7-8x^4+4x^3-7=0$$

(f) A relation ρ is defined on the set Z by "$a\rho b$ iff $ab>0$" for $a,b\in Z$. Examine if ρ is reflexive and transitive (where Z denotes the set of integers).

(g) Find the eigen values of the matrix $A=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}$ and verify Cayley-Hamilton theorem for A.

(h) Let ρ be an equivalence relation on a set S and $a,b\in S$. If $a\bar{\rho}b$, then cl(a) and cl(b) are disjoint.

2. (a) Find the roots of the equation $z^n=(z+1)^n$, where n (> 1) is a positive integer. Show that the points which represent them in the z-plane are collinear. 3

(b) Solve the equation $x^4-x^3+2x^2-x+1=0$ which has four distinct roots of equal moduli. 3

(c) If α, β, γ are the roots of the equation $x^3+qx+r=0$, then find the value of $\sum\frac{1}{\alpha^2-\beta\gamma}$ 2

3. (a) For a suitable value of h, apply the transformation $x = y + h$ to remove the term of x^2 from the equation $x^3 - 15x^2 - 33x + 847 = 0$, and then solve the transformed equation by Cardan's method. Hence, find the roots of the given equation. 5

(b) If a, b, c, d are positive real numbers such that $a+b+c+d=1$, prove that $\frac{a}{1+b+c+d}+\frac{b}{1+c+d+a}+\frac{c}{1+d+a+b}+\frac{d}{1+a+b+c}\geq\frac{4}{7}$ 3

4. (a) Prove that $R=\{(a,b)\in\mathbb{Z}\times\mathbb{Z}:\ 8a+5b$ is divisible by $18\}$ is an equivalence relation on the set of integers $\mathbb{Z}$. 3

(b) Prove that a reflexive relation ρ on a nonempty set S is an equivalence relation on S if and only if $(a,b)\in\rho$ and $(b,c)\in\rho$ imply that $(c,a)\in\rho$ for any $a,b,c\in S$. 2

(c) Let R be an equivalence relation on a set S and for $a\in S$, let $[a]$ denote the R-equivalence class of a in S. For any two elements $x,y\in S$ if $[x]\neq[y]$, show that $[x]\cap[y]=\phi$. 3

5. (a) Let $f,\, g:\mathbb{R}\to\mathbb{R}$ be two functions given by $f(x)=|x|+x$ for all $x\in\mathbb{R}$ and $g(x)=|x|-x$ for all $x\in\mathbb{R}$. Find $f\circ g$ 3

(b) For two nonempty sets X and Y, let $f:\ X\to Y$ be a mapping such that $f(A\cap B)=f(A)\cap f(B)$ for all nonempty subsets A and B of X. Prove that f is injective. 2

(c) Show that the open intervals $(0, 1)$ and $(0, \infty)$ are of same cardinality. 3

6. (a) State Well-ordering property of positive integers. Also state Fundamental Theorem of Arithmetic. 2

(b) Use mathematical induction to prove that for any positive integer n, 7 divides $3^{2n+1}+2^{n+2}$ 3

(c) Show that there are infinitely many primes of the form $4n+3$, n being non-negative integers. 3

7. (a) Use Chinese remainder theorem to solve: 3

$$x\equiv 2 \pmod 7$$
$$x\equiv 3 \pmod 9$$
$$x\equiv 2 \pmod{11}.$$

(b) Solve the congruence $12x\equiv 9 \pmod{15}$. 3

(c) Find $\phi(260)$, where ϕ stands for Euler's phi-function. 1

(d) Determine that integers $n\geq 3$ such that $5\equiv n \pmod{n^2}$. 1

8. (a) Determine the third degree polynomial function $f(x)=ax^3+bx^2+cx+d$ whose graph passes through the points (– 1, 1), (1, 1), (2, – 2) and (3, 1). 3

(b) Determine if the following system is consistent: 3

$$x_2-4x_3=8$$
$$2x_1-3x_2+2x_3=1$$
$$5x_1-8x_2+7x_3=1$$

(c) Find the value of k for which the system of equations $kx+y+z=1$, $x+ky+z=1$, $x+y+kz=1$ will have a unique solution. 2

9. (a) Find the rank of the following matrix when λ lies in the open interval (–1, 2). 2

$$\begin{bmatrix} \lambda & 1 & 0 \\ 3 & \lambda-2 & 1 \\ 3(\lambda+1) & 0 & \lambda+1 \end{bmatrix}$$

(b) Show that the matrix $A=\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ has a 2-fold eigenvalue. Determine the set of all eigenvectors corresponding to that eigenvalue of A. 3

(c) State Cayley-Hamilton Theorem and verify it for the matrix $A=\begin{pmatrix} 0 & 0 & -90 \\ 1 & 0 & 39 \\ 0 & 1 & 0 \end{pmatrix}$ 3

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